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Attrition Modeling in the Presence of Decoys: An Operations-Other-Than-War Motivation

by

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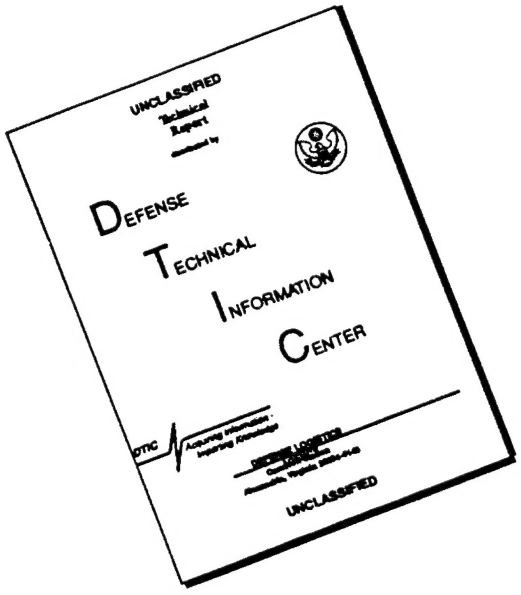

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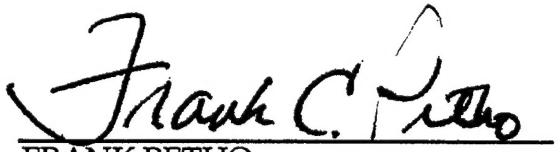

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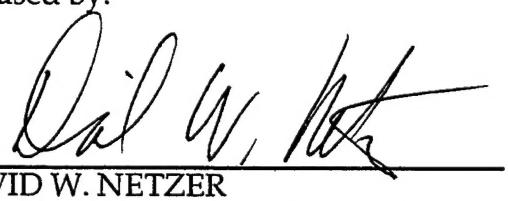

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Attrition Modeling in the Presence of Decoys: An Operations-Other-Than-War Motivation

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Abstract

Under various operational conditions, in particular in operations other than war (OOTW) or peacekeeping, an intervening force, here Blue, must occasionally engage in attrition warfare with an opposing force, here Red, that is intermingled with non-combatants. Desirably, Red armed actives are targeted and not the unarmed non-combatants. This paper describes some simple Lanchesterian attrition models that reflect a certain capacity of Blue to discriminate non-combatants from armed and active Red opponents. An explicit extension of the "Lanchester square law" results: Blue's abstinence concerning the indiscriminate shooting of civilians mixed with Reds is essentially reflected in a lower Blue rate of fire and less advantageous exchange rate. The model applies to other situations involving decoys, and reflects the value of a discrimination capability.

1. Introduction

Mutual attrition on the battlefield has classically been modeled without accounting for the possible presence of false targets: decoys of low military value intended to divert opponent fire, or even the deliberate dispersal of unarmed civilians among armed and active combatants. The latter is a situation that might

well occur in the operations other than war (OOTW) scenarios anticipated as one of the Major Regional Contingencies (MRCs) types into which the U.S. or joint forces could be drawn.

The issue to be addressed herein is that of understanding how Red's use of decoys, e.g. civilians, for cover, influences Blue's capability to inflict attrition upon Red armed active forces, and at what expense in terms of Blue's own attrition and the inadvertent attrition inflicted, or wasted, upon such decoys. Clearly attrition of human decoys is to be strenuously avoided for humanitarian reasons, but also because of its broad impact on world opinion; in some circumstances such attrition might well inflame resistance to the extent that the civilian population could itself become an active threat. But in order for Blue to avoid killing Red-controlled civilians, or less politically-sensitive targets, i.e. to avoid wasting time and resources that could otherwise be directed towards targeting Red actives, some sacrifice in Blue effectiveness must be accepted.

We provide here a preliminary set of simple models for quantifying the effect of substituting discrimination for pure attrition when false targets are present. It will be seen that the effect of accounting for target discrimination power by Blue can be reduced to an explicit formula that generalizes the classical Lanchesterian "square law". Elaboration to include more realistic detail induces the need for more ambitious numerical work, but the latter is not formidable. Addition of Blue various force and Red decoy types, Blue (inanimate/non-human) decoys, stochastics, and the aforementioned change of affiliation by Red (or Blue: slaughter of Red civilians by Blue forces may induce the latter to either slacken their attack, or stimulate greater Red resistance) can all be modeled and ultimately quantified.

2. Initial, and Simplest, Formulation

Let

$R_a(t)$ = number of Red active, attrition-capable, armed military forces at time t;

$R_c(t)$ = number of Red unarmed civilians or other decoys mixed with the above at t;

$B(t)$ = number of Blue active armed forces at t.

In what follows it is clearly necessary to require $B(t) \geq 0$, $R(t) \geq 0$; otherwise nonsense results occur. The attrition equations are non-linear.

Mechanism of combat: the Red actives deplete the Blue actives according to aimed-fire (square-law) Lanchester

$$\frac{dB}{dt} = -\rho_{RB}(t)R_a(t). \tag{2.0}$$

The Blue actives attempt to do the same, but must avoid killing civilians, or generally being diverted by decoys.

2.1 Blue Shoots at First Available Red.

If a Blue simply picks a target Red at random then, assuming military and civilians are well mixed and appear to the Blues in proportion to their numbers, he/she targets an active Red with success probability $s_a(t) = R_a(t)/[R_a(t) + R_c(t)]$, so

$$\frac{dR_a}{dt} = -\rho_{BR}(t)s_a(t)B(t) = -\rho_{BR}(t)\left[\frac{R_a(t)}{R_a(t)+R_c(t)}\right]B(t). \tag{2.1}$$

Note that it is to Blue's immediate selfish advantage to discriminate between Red actives and civilians, for in the above $R_a(t)/[R_a(t)+R_c(t)]$ can well be considerably less than unity, in which case Blue only slowly reduces those shooting at him/her. Red civilians are targeted with probability $1 - s_a(t) = s_c(t)$; the results may be quite unacceptable from Blue's viewpoint, and certainly from that of Red. This is a low-resolution model: differentiation between Blue force

types, and coordination of fire capabilities, are important in practice but not addressed here. The payoff is a rather explicit analytical result.

2.2 Blue Possesses Discriminatory Powers.

Suppose Blue discriminates between Red actives and Red civilians: i_{aa} is the probability that Blue can identify a Red active if he acquires one; i_{ac} is the probability that he mis-identifies it as a decoy (civilian); i_{ca}, the probability that a decoy (civilian) is mistaken for an active, and i_{cc}, the probability of correct identification of a decoy (civilian), are defined correspondingly. We hope that i_{aa} and i_{cc} are near unity, but there may be a substantial cost in time for this capability. Now $R_a(t)i_{aa}$ is the (approximate) number of Red actives correctly identified at t, while $R_a(t)i_{ac}$ is the number incorrectly classed as civilians and not shot at, and $R_c(t)i_{ca}$ is the number of civilians targeted through misclassification error. Then $R_a(t)i_{aa}/[R_a(t)i_{aa} + R_a(t)i_{ac} + R_c(t)i_{ca} + R_c(t)i_{cc}]$ is the fraction (of time dt) spent correctly shooting at Red actives, so

$$\begin{aligned}\frac{dR_a(t)}{dt} &= -\rho_{BR}(t)\frac{R_a(t)i_{aa}(t)}{R_a(t)(i_{aa}+i_{ac})+R_c(t)(i_{ca}+i_{cc})}B(t)\\ &= -\rho_{BR}(t)\frac{R_a(t)i_{aa}(t)B(t)}{R_a(t)+R_c(t)};\end{aligned} \tag{2.2}$$

the above holds because $i_{aa} + i_{ac} = 1$, $i_{ca} + i_{cc} = 1$. Notice that the identification probabilities i_{cc}, etc., could be made time-dependent to represent changes in visibility throughout the conflict. We can also write the attrition equation for decoys, or civilians:

$$\frac{dR_c}{dt} = -\rho_{BR}(t)\frac{R_c(t)i_{ca}(t)}{R_a(t)+R_c(t)}B(t). \tag{2.3}$$

The above formulations assume that the Red actives and civilians are well-mixed and hence equally likely to be found by a Blue active; however, once found, a candidate target can be assessed for relevance, but with error. For now we slough

off the time-consuming aspects of this process. A model with more states can handle this aspect; see Section 3.

To move towards actual solutions, divide (2.2) by (2.3):

$$\frac{\frac{dR_a}{dt}}{\frac{dR_c}{dt}} = \frac{-\rho_{BR}(t)\frac{R_a(t)i_{aa}(t)}{R_a(t)+R_c(t)}B(t)}{-\rho_{BR}(t)\frac{R_c(t)i_{ca}(t)}{R_a(t)+R_c(t)}B(t)} = \frac{R_a(t)i_{aa}(t)}{R_c(t)i_{ca}(t)}, \tag{2.4}$$

so upon division

$$\frac{(dR_a/dt)}{R_a(t)} \Big/ \frac{(dR_c/dt)}{R_c(t)} = i_{aa}(t)/i_{ca}(t). \tag{2.5}$$

For simplicity drop the i-time-dependency; easy explicit integration gives:

$$\ln R_a(t) - \ln R_a(0) = \left(\frac{i_{aa}}{i_{ca}}\right)\left(\ln R_c(t) - \ln R_c(0)\right). \tag{2.6}$$

From this,

$$R_c(t) = R_c(0)\cdot\left(R_a(t)/R_a(0)\right)^{i_{ca}/i_{aa}}. \tag{2.7}$$

Now plug this into equation (2.2):

$$\frac{dR_a}{dt} = -\rho_{BR}(t)\frac{R_a(t)i_{aa}B(t)}{R_a(t)+R_c(0)\left(R_a(t)/R_a(0)\right)^{i_{ca}/i_{aa}}}. \tag{2.8}$$

Equations (2.0) and (2.8) constitute a pair of non-linear first-order differential equations that can be routinely solved numerically, subject to initial and boundary conditions: $0 \le R_a(t), B(t) \le R_a(0), B(0)$. Closed-form analytical solutions are practically inaccessible. Divide (2.0) by (2.8): we come up with an equation that relates $B(t)$ and $R_a(t)$ *that can be integrated explicitly.* We anticipate that a "generalized square law" will show itself (no disappointment here!).

Proceed to solve for R_a in terms of B:

$$\frac{dR_a/dt}{dB/dt} = \frac{-\rho_{BR}(t)\dfrac{R_a(t)i_{aa}B(t)}{R_a(t)+\left(R_c(0)/R_a(0)^{i_{ca}/i_{aa}}\right)\left(R_a(t)\right)^{i_{ca}/i_{aa}}}}{-\rho_{RB}(t)R_a(t)}. \tag{2.9}$$

Rearranging,

$$\left(R_a(t)+\left(R_c(0)/\left(R_a(0)\right)^{i_{ca}/i_{aa}}\right)\left(R_a(t)\right)^{i_{ca}/i_{aa}}\right)dR_a(t) = \frac{\rho_{BR}(t)}{\rho_{RB}(t)}i_{aa}B(t)dB(t). \tag{2.10}$$

Assume that $\rho_{BR}(t)/\rho_{RB}(t)$ is independent of t and integrate to get, finally,

$$\begin{aligned}&\frac{R_a^2(t)}{2}-\frac{R_a^2(0)}{2}+\left(\frac{R_c(0)}{\left(R_a(0)\right)^{i_{ca}/i_{aa}}}\right)\left[\frac{R_a(t)^{(i_{ca}/i_{aa})+1}}{(i_{ca}/i_{aa})+1}-\frac{R_a(0)^{(i_{ca}/i_{aa})+1}}{(i_{ca}/i_{aa})+1}\right]\\ &=\frac{\rho_{BR}}{\rho_{RB}}i_{aa}\left[\frac{B^2(t)}{2}-\frac{B^2(0)}{2}\right].\end{aligned} \tag{2.11}$$

This is a "generalized square law", valid when positive boundary conditions are respected. Notice that if $R_c(0) = 0$ we are back to the original square law, immortalized in song and story; see for instance Taylor (1980). If $R_c(0) > 0$ but $i_{ca} = i_{aa}$ we also have a new and somewhat different square law. Anderson (1995) has also considered the influence of decoys or false targets in a more elaborate setting. His formulation differs from ours in that discrimination capability is not represented by functions i_{aa}, i_{ca} (and their respective complements).

Illustration 1: $i_{ca} = i_{aa}$, $R_c(0) = R_a(0)$

This is a pessimistic case for Blue, who has no discriminatory power. But the result is simple:

$$\frac{R_a^2(t)}{2} - \frac{R_a^2(0)}{2} + \left[\frac{R_a^2(t)}{2} - \frac{R_a^2(0)}{2}\right] = \frac{\rho_{BR}}{\rho_{RB}} i_{aa} \left[\frac{B^2(t)}{2} - \frac{B^2(0)}{2}\right]$$

or (2.12)

$$R_a^2(0) - R_a^2(t) = \left(\frac{\rho_{BR}}{\rho_{RB}}\right)\left(\frac{i_{aa}}{2}\right)\left[B^2(0) - B^2(t)\right],$$

which is a new square-law result.

The above is precisely the same equation that would occur if there were no civilians ($R_c(0) = 0$), but with ρ_{BR}, the attrition rate of Blue *vs.* Red, replaced — *reduced* — to $\rho_{BR}\, i_{aa}/2$. In this case the presence of civilians or decoys has diluted the Blue force's effectiveness by $i_{aa}/2$, i.e. by at least a factor of two. Furthermore, civilians are still getting targeted and presumably killed since $i_{ca} = i_{aa}$, which, it is hoped, is unrealistic in practice. This disadvantage must be overcome by sharpening Blue's perception so as to reduce i_{ca} well below i_{aa}, which would allow return (nearly) to classical attrition formulas. Otherwise, more Blue forces would be needed to achieve desired results.

Illustration 2: $i_{ca} = 0$

This is optimistic for Red civilian non-combatants: they are never targeted. Note that (2.11) becomes

$$\frac{R_a^2(0)}{2} - \frac{R_a^2(t)}{2} + R_c(0)\left[R_a(0) - R_a(t)\right] = \frac{\rho_{BR}}{\rho_{RB}} i_{aa} \left[\frac{B^2(0)}{2} - \frac{B^2(t)}{2}\right]. \qquad (2.13)$$

From this it is apparent that surviving Red attacker number, $R_a(t)$, increases with $R_c(0)$, initial decoy supply, as is intuitive: the presence of Red decoys still interferes with Blue's effectiveness, even though none are actually engaged.

Numerical Examples

Figures 1 – 3 present the results of numerically solving equations (2.2) – (2.3). The MATLAB, version 4.0, 4^{th} and 5^{th} order Runge-Kutta-Fehlberg numerical integration method was used. The MATLAB plotting algorithms were used to produce the figures.

In Figures 1 – 3 the initial number of active Reds, $R_a(0) = 100$; the initial number of civilian Reds, $R_c(0) = 50$; and the initial number of Blue $B(0) = 100$. In all the Figures Blue has perfect classification; that is, $i_{aa} = i_{cc} = 1$. In Figure 1, $\rho_{RB} = \rho_{BR} = 0.1$. In Figure 2, $\rho_{RB} = 0.1$, $\rho_{BR} = 0.19$. In Figure 3, $\rho_{RB} = 0.1$, $\rho_{BR} = 0.20$. Note that even though Blue has perfect classification capability, the effort that it expends doing the classification puts Blue at a disadvantage when $\rho_{RB} = \rho_{BR} = 0.1$. Blue needs to almost double ρ_{BR} to achieve parity with Red.

In Figures 4 – 5, $R_a(0) = 100$; $R_c(0) = 50$; $\rho_{RB} = \rho_{BR} = 0.1$; and $i_{aa} = i_{cc} = 1$. In order to achieve parity with Red, the initial Blue forces must be nearly 145, a 40% increase.

In Figures 6 – 7, $R_a(0) = 100$; $R_c(0) = 50$; $\rho_{RB} = \rho_{BR} = 0.1$; and $i_{cc} = 1$. However, $i_{aa} = 0.8$. Thus, Blue can mistake active Reds for civilians. In order to achieve parity with Red in this case, the initial number of Blues needs to be almost 160.

Table 1 displays the times until 85% of the initial Red Actives are attrited and times until 85% of the Blue forces are attrited. In all cases the initial number of Red actives is 100 and the initial number of Red civilians is 50. The parameters $\rho_{RB} = \rho_{BR} = 0.1$ Blue always classifies Red civilians correctly; $i_{cc} = 1$. The side that reaches 85% of its initial forces first is considered the loser. Even if Blue has

perfect classification of active Reds, $i_{aa} = 1$, Blue needs more than 20 additional forces to be the winner when there are 50 Red civilians in the area. If further, Blue does not have perfect classification, $i_{aa} = 0.8$, then Blue needs more than 30 additional forces to be the winner.

Table 1. Time Until 85% of Force is Attrited
50 Red Civilians
$\rho_{BR} = \rho_{RB} = 0.1$

	Initial Forces		85% of Initial Forces		Time Until 85% of Initial Force Are Attrited		
i_{aa}	Red Actives	Blue	Red Actives	Blue	Red Actives	Blue	Winner
1	100	100	85	85	2.6	1.6	RED
1	100	110	85	93.5	2.3	1.8	RED
1	100	120	85	102	2.1	1.9	RED
1	100	130	85	110.5	1.9	2.2	BLUE
0.85	100	130	85	110.5	2.4	2.1	RED
0.85	100	140	85	119	2.2	2.3	BLUE

To compensate for likely variability of outcome and uncertainty, Blue force size to guarantee winning will very likely be much larger than in the above table.

3. Alternative Seeker-Attriter Conflict Formulations

A more realistic, but also more complex problem formulation explicitly distinguishes between blue Seekers and Blue Actives: the former locate Red units, while the latter attack/attrit those Reds detected and then identified (possibly incorrectly) as Actives. We define

$R_{af}(t)$ = number of Red actives that are free (undetected by Blue) at time t;
$R_{ad}(t)$ = number of Red actives that are detected by Blue at time t;
$R_{cf}(t)$ = number of Red civilians (or decoys) free at time t;
$R_{cd}(t)$ = number of Red civilians (or decoys) detected (erroneously) at t;
$B_s(t)$ = number of Blue seekers at time t;
$B_a(t)$ = number of Blue actives at time t.

Various issues add to the modeling choices possible; we explore examples only. Specifically, here Blue Seekers are modeled as *detectors* of Reds, but not as follower-trackers: once a Blue Seeker makes contact with a Red it puts it into a "detected pool" that is available for Blue Active attack, but the Blue Seeker becomes quickly free again. A plausible alternative is that a Blue Seeker identifies and labels and binds to a Red until the latter is attacked or released. We arbitrarily omit consideration of this option for the present.

There follow the state transition equations.

$$\frac{dR_{af}}{dt} = \underbrace{-\xi_{BR}(a)R_{af}(t)B_s(t)}_{\substack{\text{Free Active Reds Detected} \\ \text{by Blue Seekers}}} + \underbrace{\lambda_{df}(a)R_{ad}(t)}_{\substack{\text{Detected Active Reds'} \\ \text{Track Lost (freed)}}} \tag{3.1}$$

$$\frac{dR_{ad}}{dt} = \underbrace{\xi_{BR}(a)R_{af}(t)B_s(t)}_{\text{Free Active Reds Detected}} - \underbrace{\lambda_{df}(a)R_{ad}(t)}_{\substack{\text{Detected Active} \\ \text{Reds freed}}} - \underbrace{\rho_{BR}\frac{R_{ad}(t)i_{aa}}{R_{ad}(t)+R_{cd}(t)}B_a(t)}_{\substack{\text{Detected Active} \\ \text{Reds Attrited}}} \tag{3.2}$$

$$\frac{dR_{cf}(t)}{dt} = \underbrace{-\xi_{BR}(c)R_{cf}(t)B_s(t)}_{\substack{\text{Civilian/Decoy} \\ \text{Red Detected}}} + \underbrace{\lambda_{df}(c)R_{cd}(t)}_{\substack{\text{Civilian/Decoy} \\ \text{Red Track Lost (freed)}}} \tag{3.3}$$

$$\frac{dR_{cd}(t)}{dt} = \underbrace{\xi_{BR}(c)R_{cf}(t)B_s(t)}_{\substack{\text{Civilian/Decoy} \\ \text{Red Detected}}} - \underbrace{\lambda_{df}(c)R_{cd}(t)}_{\substack{\text{Civilian/Decoy} \\ \text{Red Track Lost}}} - \underbrace{\rho_{BR}\frac{R_{cd}(t)i_{ca}}{R_{ad}(t)+R_{cd}(t)}B_a(t)}_{\substack{\text{Detected Civilian/Decoy} \\ \text{Red Attrited (Mistakenly)}}} \tag{3.4}$$

$$\frac{dB_s(t)}{dt} = \underbrace{-\rho_{RB}(s)\left(R_{af}(t)+R_{ad}(t)\right)}_{\substack{\text{Blue Seeker attrited by} \\ \text{Red Actives}}} \tag{3.5}$$

$$\frac{dB_a(t)}{dt} = \underbrace{-\rho_{RB}(a)\left(R_{af}(t)+R_{ad}(t)\right)}_{\substack{\text{Blue Active attrited by} \\ \text{Red Actives}}} \tag{3.6}$$

The condition that all state variables be non-negative must be respected when solving (3.1) – (3.6); numerical solution is the only practical option. The rate parameters are self-explanatory; if desired any or all of these may be made time-dependent.

4. Summary Discussion

The simple models proposed above account in an explicit way for a significant combat phenomenon: the influence of false targets or decoys upon a (Blue) combatant's attritional effectiveness. They incorporate discrimination power (probability) parameters (i_{aa}, i_{cc}) that represent the capacity of Blue to correctly identify Red decoys, or, in another interpretation, the capacity of Red to confuse and divert Blue. It is hoped that our paper will stimulate further investigation in this important military area, which is of considerable current interest: it investigates a specific aspect of what is presently called Information Warfare.

A referee has observed a similarity between this paper's models and those previously introduced by Taylor (1980), and subsequently discussed by Roberts and Conolly (1992). The similarity is that the latter model also stipulates two force types (analogous to our Red, however, for them both are active and attrition-capable), vs. one (our Blue). But our emphasis differs in that explicit attention is paid to the uncertainty with which the single force (our Blue) can actually determine *which* Red force type element is currently a possible target. Discrimination parameters (alternatively, a confusion matrix) are introduced to explicitly represent the degree with which the Blue force can avoid being tricked by Red. This feature might desirably be introduced into the Taylor-Roberts and Conolly models; as they now stand (see (1) of Roberts and Conolly) their formulation takes no explicit account of which Red type Blue is likely to be

targeting; on a modern battlefield this could be an important issue. Optimization could be carried out with respect to real-time targeting as well as by choice of initial force size.

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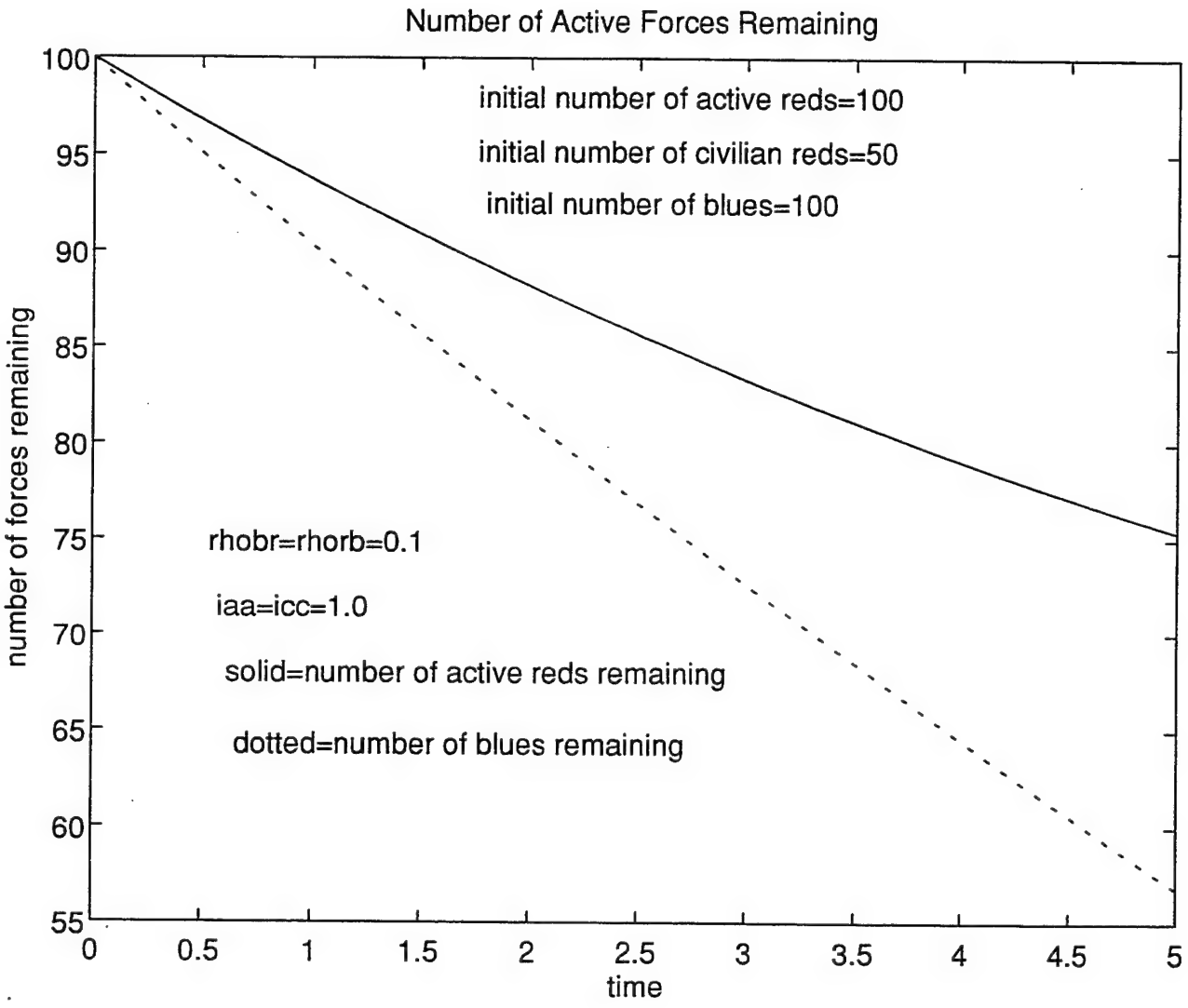
Number of Active Forces Remaining
initial number of active reds=100
initial number of civilian reds=50
initial number of blues=100
rhobr=rhorb=0.1
iaa=icc=1.0
solid=number of active reds remaining
dotted=number of blues remaining
number of forces remaining
time
100
95
90
85
80
75
70
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0.5
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1.5
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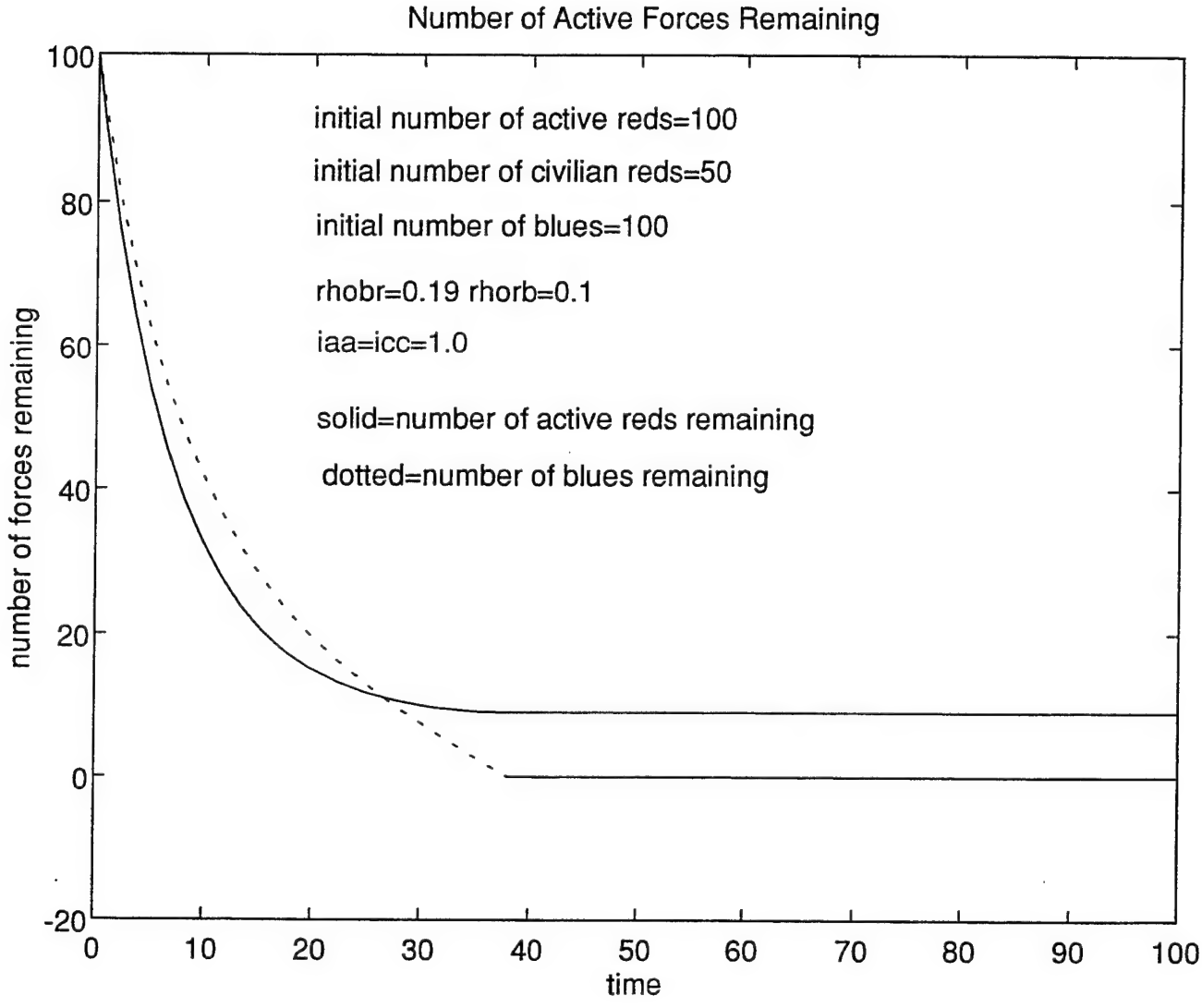
Figure 1


Number of Active Forces Remaining
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initial number of civilian reds=50
initial number of blues=100
rhobr=0.19 rhorb=0.1
iaa=icc=1.0
solid=number of active reds remaining
dotted=number of blues remaining
number of forces remaining
time
100
80
60
40
20
0
-20
0
10
20
30
40
50
60
70
80
90
100

Figure 2

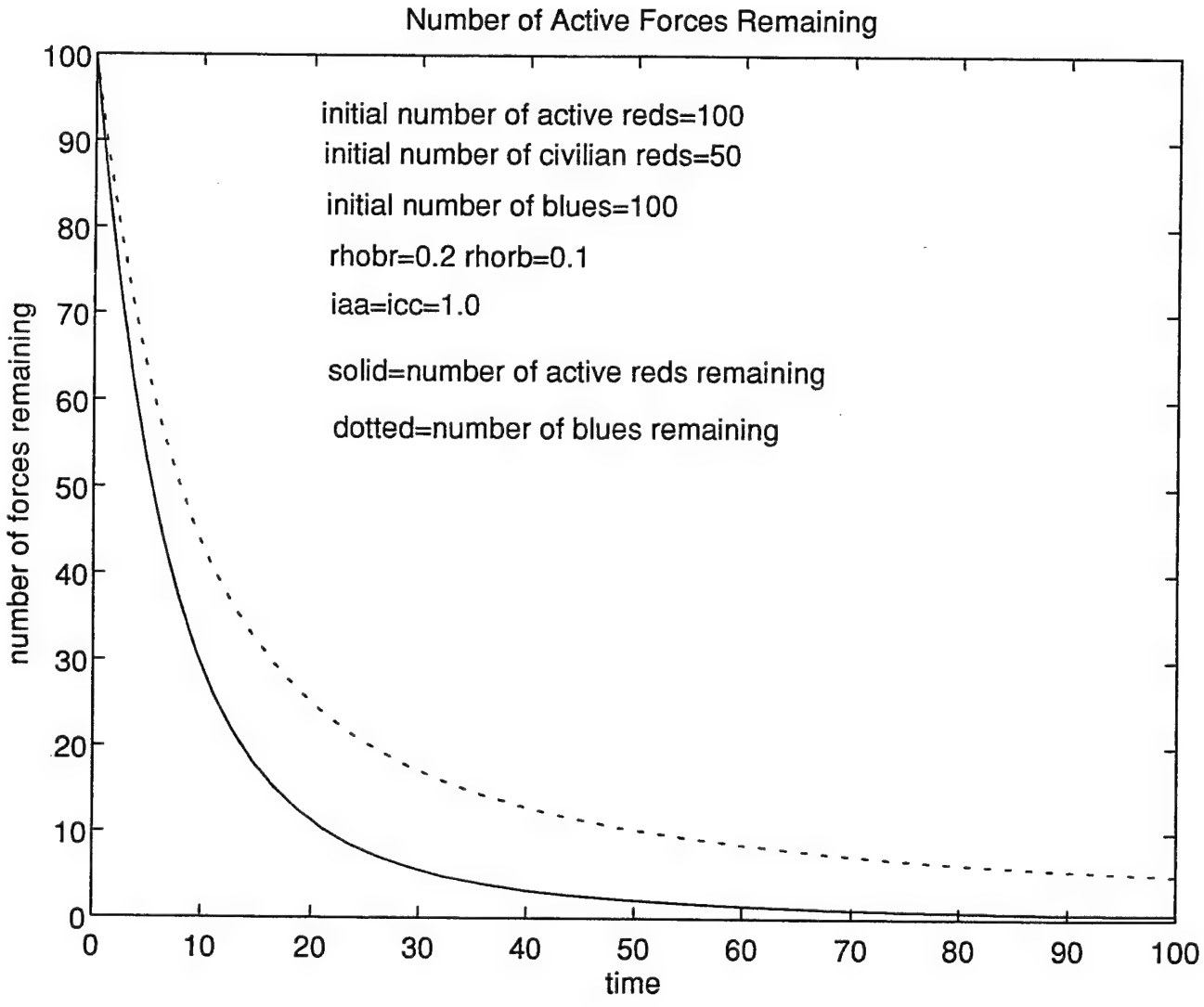
Number of Active Forces Remaining
initial number of active reds=100
initial number of civilian reds=50
initial number of blues=100
rhobr=0.2 rhorb=0.1
iaa=icc=1.0
solid=number of active reds remaining
dotted=number of blues remaining
number of forces remaining
time
0
10
20
30
40
50
60
70
80
90
100

Figure 3

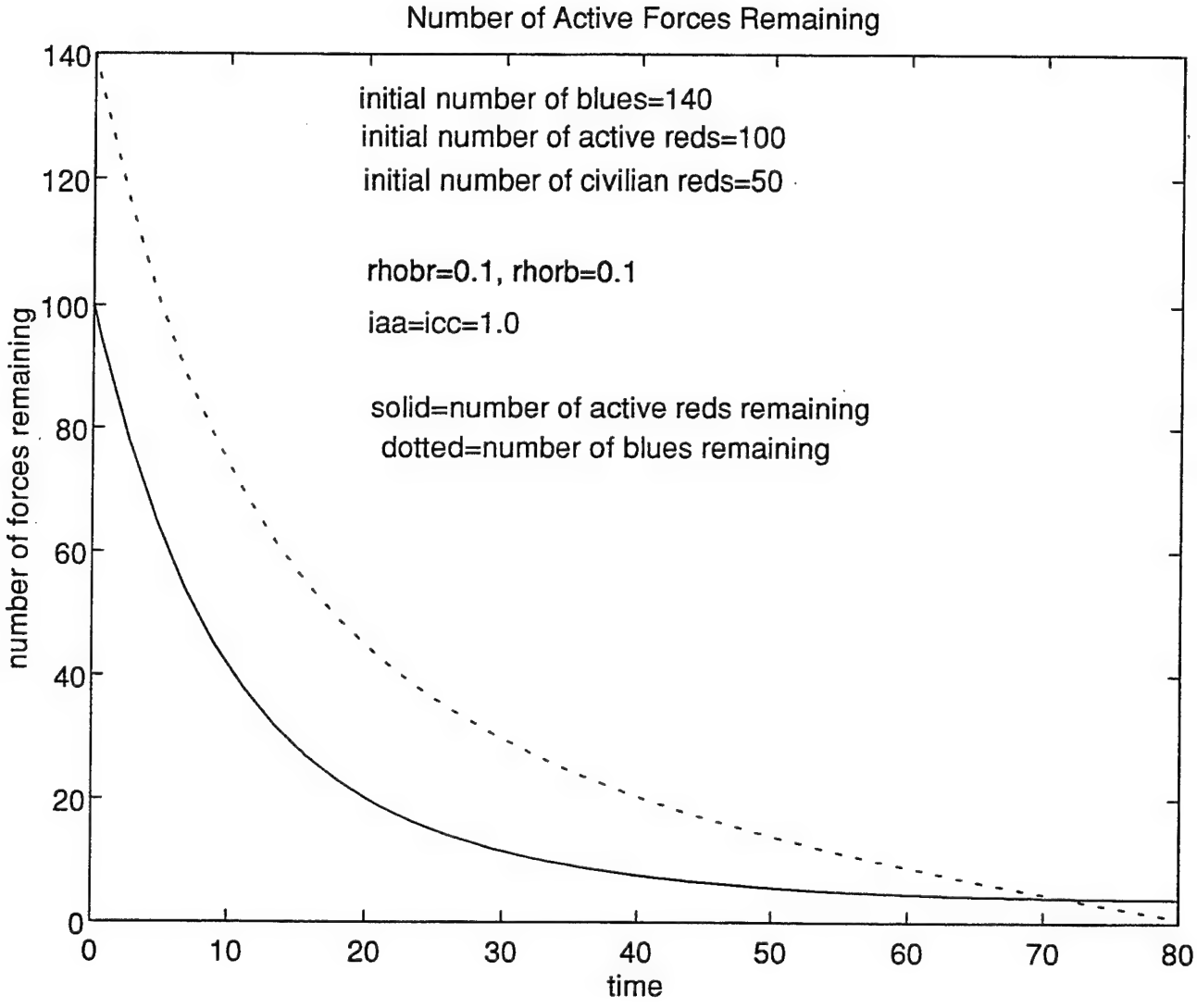
Number of Active Forces Remaining
initial number of blues=140
initial number of active reds=100
initial number of civilian reds=50
rhobr=0.1, rhorb=0.1
iaa=icc=1.0
solid=number of active reds remaining
dotted=number of blues remaining
number of forces remaining
time
0
20
40
60
80
100
120
140
0
10
20
30
40
50
60
70
80

Figure 4

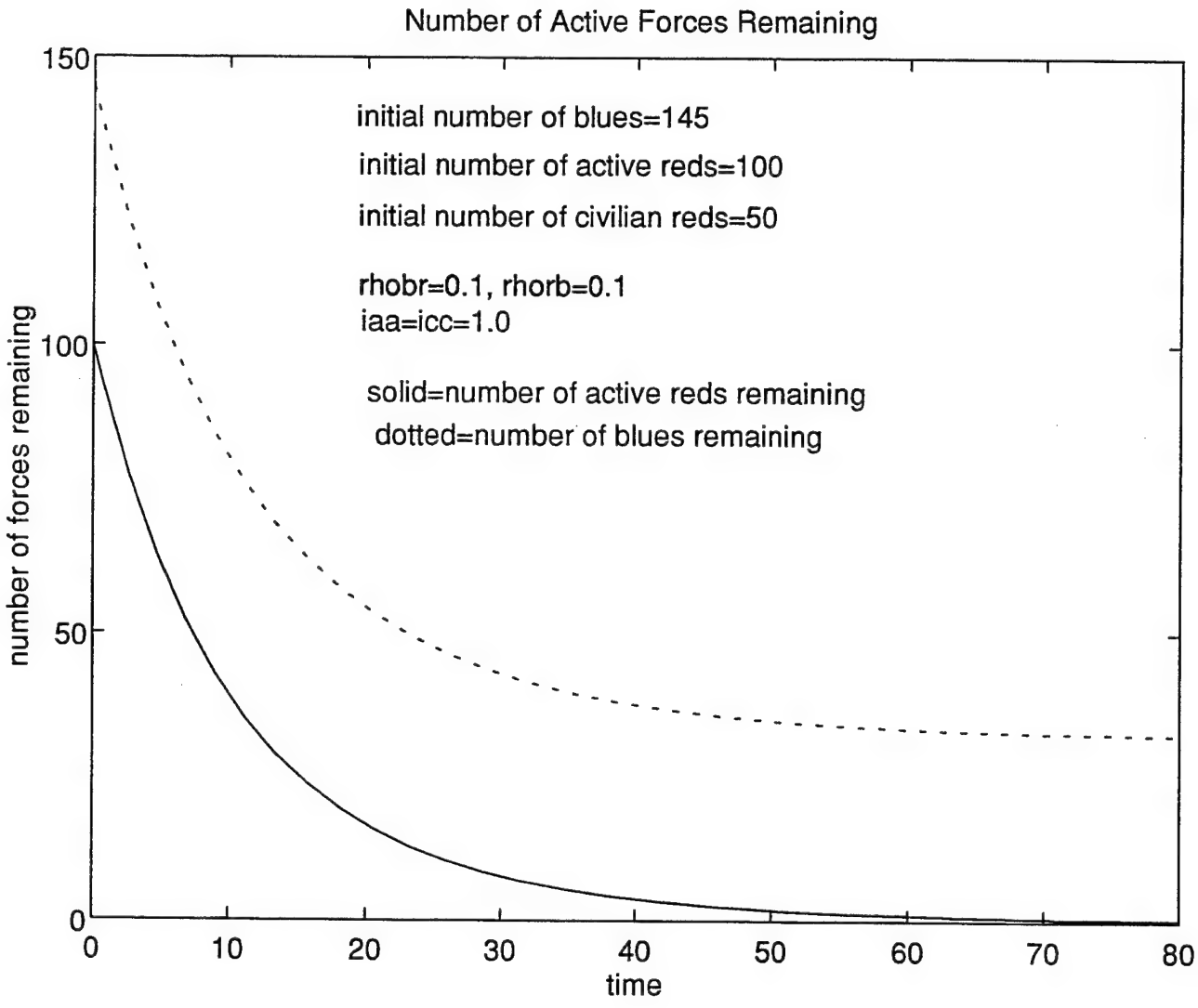
Number of Active Forces Remaining
initial number of blues=145
initial number of active reds=100
initial number of civilian reds=50
rhobr=0.1, rhorb=0.1
iaa=icc=1.0
solid=number of active reds remaining
dotted=number of blues remaining
number of forces remaining
time
150
100
50
0
0
10
20
30
40
50
60
70
80

Figure 5

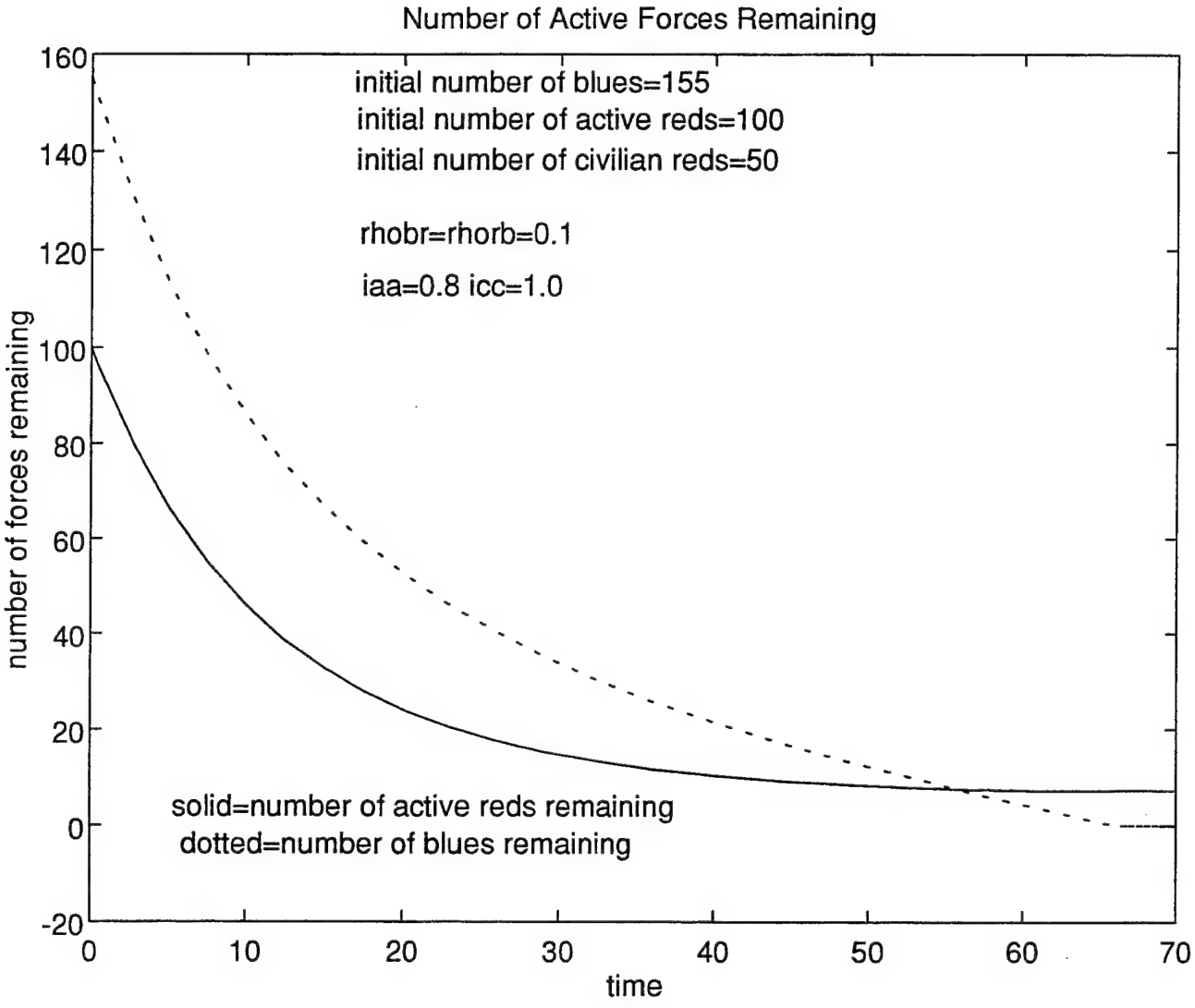
Number of Active Forces Remaining
initial number of blues=155
initial number of active reds=100
initial number of civilian reds=50
rhobr=rhorb=0.1
iaa=0.8 icc=1.0
solid=number of active reds remaining
dotted=number of blues remaining
number of forces remaining
time
160
140
120
100
80
60
40
20
0
-20
0
10
20
30
40
50
60
70

Figure 6

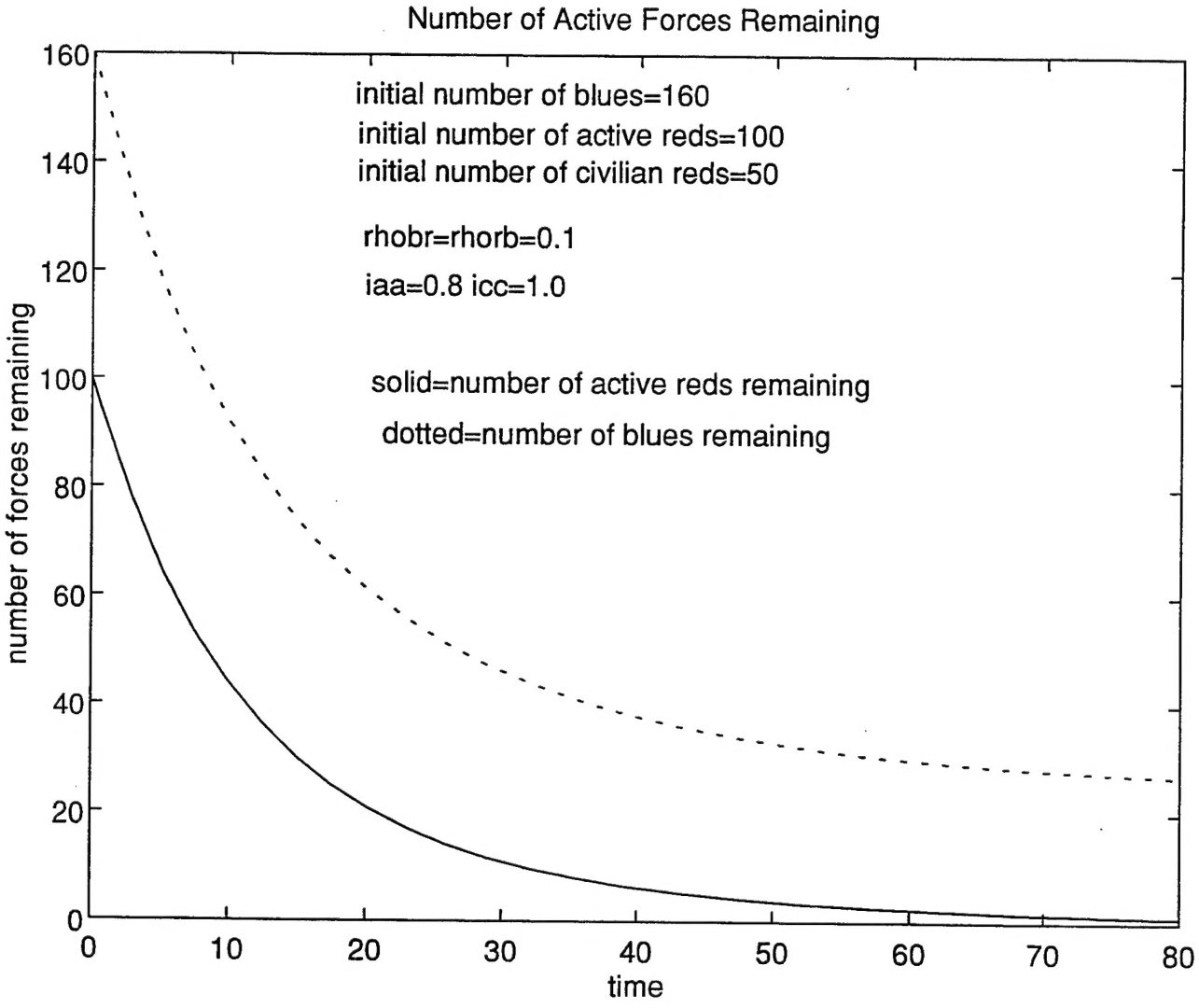
Number of Active Forces Remaining
initial number of blues=160
initial number of active reds=100
initial number of civilian reds=50
rhobr=rhorb=0.1
iaa=0.8 icc=1.0
solid=number of active reds remaining
dotted=number of blues remaining
number of forces remaining
time
0
20
40
60
80
100
120
140
160
0
10
20
30
40
50
60
70
80

Figure 7

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